Liquid Crystals
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713926090

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Online publication date: 25 May 2010

To cite this Article De Smet Corresponding author, Herbert , Steen Corresponding author, Jean Van den and Cuypers Corresponding author, Dieter(2004) 'Electrical model of a liquid crystal pixel with dynamic, voltage history-dependent capacitance value', Liquid Crystals, 31: 5, 705 — 711

To link to this Article: DOI: 10.1080/02678290410001675129

URL: http://dx.doi.org/10.1080/02678290410001675129

LIQUID CRYSTALS, VOL. 31, NO. 5, MAY 2004, 705–711

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Electrical model of a liquid crystal pixel with dynamic, voltage history-dependent capacitance value

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(*Received 25 September 2003; in final form 5 January 2004; accepted 15 January 2004*)

The voltage history-dependent nature of a liquid crystal pixel capacitance becomes an important issue in active matrix addressing as large storage capacitors are replaced with more intelligent circuitry such as in-pixel frame buffers. In this paper, a simple but flexible Spice macro model is introduced that allows accurate simulations of the electrical behaviour of a nematic liquid crystal pixel capacitance. The model correctly predicts voltage drops caused by the increasing dielectric constant when liquid crystal molecules align themselves to the electric field. An internal node of the macro model represents the average director orientation and can also be used to predict the optical response. In its basic embodiment, the model uses a first-order, low pass RMS filter to implement the dynamic behaviour of the pixel, which suffices to predict response delays and asymmetric rise and fall times. However, the model also supports more elaborated filters that offer more control over the simulated dynamic behaviour. A number of simulations are performed that illustrate the usefulness of the new model during the design of novel 'smart' pixel architectures.

1. Introduction

With the advent of liquid crystal on silicon (LCOS) displays [1], it has become feasible to incorporate several transistors under every pixel of an active matrix display. This opens the path towards what is commonly called 'smart' pixels. The most notable example of this is a pixel with integrated frame buffering functionality [2]. Such a pixel contains a memory cell, either analogue or digital, that can be used to write and store the image information for the next frame period, without influencing the voltage that is present on the pixel electrode. A common signal line can then be used to trigger the transfer of the stored information to the pixel electrode, allowing simultaneous image transitions in the whole active matrix. Frame buffering is especially attractive for addressing schemes that require rapid image transitions, such as frame sequential colour addressing.

The need for a good electrical model for a liquid crystal pixel becomes apparent during the design of smart pixels, where the most important limitation is the area constraint. As a result, only a small number of components can be harvested inside the pixel. This implies that, for example, an in-pixel frame buffer device will consist of a rather rudimentary circuit, and its operation will depend heavily on the individual component characteristics, including those of the liquid crystal pixel itself. Furthermore, putting extra transistors under a pixel reduces the area that is available for creating a large storage capacitor in parallel with the actual pixel. It is easy to see that the smaller this storage capacitor becomes, the more important the exact value of the actual pixel capacitance will be.

Although it is well known that a nematic liquid crystal pixel behaves as a variable capacitor, a good electrical model for a pixel has not yet been published to our knowledge. Some groups use an approximate empirical model that fails to resolve the time dependency of the capacitance value [3, 4]. Even Lueder's standard work on liquid crystal displays [5] derives the differential equation for a TFT pixel assuming a constant pixel capacitance value, and then applies the voltage-dependent nature of this capacitance to the resulting solution. The difficulty in finding a good model lies in the fact that the capacitance value is not a simple function of the applied voltage, but depends on the voltage history. In this paper, we introduce a Spice macro model that acts as a voltage history-dependent capacitor that is particularly useful for modelling a liquid crystal pixel capacitance.

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Liquid Crystals ISSN 0267-8292 print/ISSN 1366-5855 online © 2004 Taylor & Francis Ltd
http://www.tandf.co.uk/journals
DOI: 10.1080/02678290410001675129

2. Physics underlying the model

The effective dielectric constant of a liquid crystal capacitor is determined by the average orientation of the director of the liquid crystal molecules, and can vary between a value $\varepsilon_\perp$ (all liquid crystal molecules aligned horizontally and hence orthogonal to the electric field) and a value $\varepsilon_\parallel$ (all liquid crystal molecules aligned vertically and hence parallel to the electric field). For the sake of simplicity, we consider only the case of liquid crystals with a positive dielectric anisotropy ($\varepsilon_\parallel > \varepsilon_\perp$). However, the resulting model will be equally suited for liquid crystals with a negative dielectric anisotropy.

The alignment layers force the molecules to line up parallel to the substrates and to the rubbing direction in the absence of an electric field. As soon as an electric field is established perpendicular to the substrates, it becomes energetically favourable for the molecules to line up parallel to the electric field because $\varepsilon_\parallel > \varepsilon_\perp$. We assume that the average director orientation can be represented by a single, one-dimensional variable 'x'. The physical meaning of x will become clear later.

In practice, three forces are in play (see figure 1):

(1). The elastic force Kx that pulls the molecules back to their resting position $x=0$ (parallel to the alignment layer).
(2). The electrical force that tries to align the molecules parallel to the field **E**. This force is proportional to $\mathbf{E}^2$.
(3). The viscosity force $\gamma\,\mathrm{d}x/\mathrm{d}t$. This force hinders any movement and is proportional to the velocity at which the molecules move.

An inertial force, caused by the mass of the moving molecules, can be neglected, and we arrive at a first-order mechanical system. Such a system is characterized by a simple time constant τ, as can be proved in the following. The equilibrium of forces in figure 1 states that:

$$cE^2 = Kx + \gamma\frac{\mathrm{d}x}{\mathrm{d}t} \tag{1}$$

where **E** is the electric field strength, c is a constant, x is a linear and scalar measure for the average director orientation, γ is the effective viscosity and K is the effective elastic constant. Assuming this one-dimensional approach is valid, the values of c, γ and K are of course related to the physical properties of the liquid crystal cell (cell thickness, elasticity tensor, rotational viscosity, anchoring force and so on), but in a way that depends on the actual LC mode. In this paper, we are not interested in this exact relationship, but it does present a challenge for further research.

In the frequency domain, we can use $p=\mathrm{j}\omega$ to replace the derivative d/dt in equation (1), and we find:

$$x = \frac{cE^2}{K+p\gamma} = \frac{\frac{cE^2}{K}}{1+p\frac{\gamma}{K}} = \frac{\frac{cE^2}{K}}{1+p\tau}. \tag{2}$$

This is the well-known first-order system with time constant $\tau=\gamma/K$ and cut-off frequency $1/2\pi\tau$. This will be the basis of our pixel model.

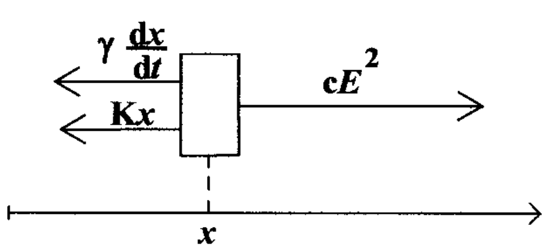

Figure 1. Simplified mechanical model for the average director orientation.

3. Voltage drop

The new model should be able to predict (*inter alia*) the voltage drop that occurs in a pixel as a result of the molecules lining up to the electric field, thereby maximizing ε. Consider the circuit shown in figure 2. The liquid crystal capacitor, which was initially at 0 V, is charged to a voltage V_0 and then disconnected from the voltage source. The electric field in the pixel will then start to align the liquid crystal molecules so that the average dielectric constant increases. As a consequence, the value of C will also increase. But as the capacitor is completely isolated, the total charge $Q=CV$ will remain the same. This means that the voltage across the pixel capacitance will decrease; this process will continue, until a steady state is reached. Graphically, it is easy to determine what the pixel voltage will be in this steady state solution, provided

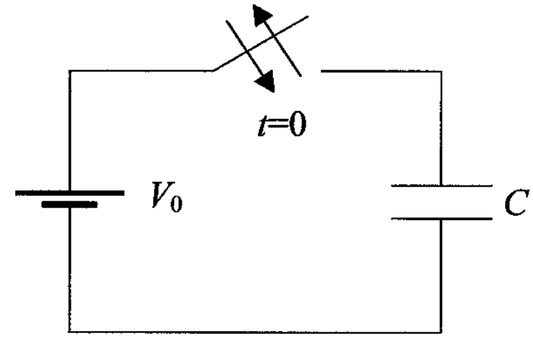

Figure 2. Experiment to illustrate the voltage drop that occurs due to the liquid crystal molecules lining up to the electric field. At $t=0$, the liquid crystal pixel is charged to an initial voltage V_0. Subsequently, the pixel capacitor is disconnected and its voltage will drop.

that we know the so-called quasi-static $C(V)$ curve of the pixel. The $C(V)$ curve gives the steady state capacitance of the pixel as a function of applied voltage. A typical $C(V)$ curve is also shown in figure 3. Usually, there is a sharp threshold at the so-called threshold voltage V_T of the pixel. In the same figure, the voltage drop problem is graphically solved. The initial situation is indicated by a first round marker: $(V_0,C_0)=(V_0,C_\perp)=(3.2\,\text{V},\ 2.5\,\text{fF})$. By definition, the steady state solution (V_∞, C_∞) will be situated on the quasi-static $C(V)$ curve. Hence $C_\infty = C(V_\infty)$. Because the total charge $Q=CV$ has to remain constant, the solution for $t=\infty$ will be given by

$$V_\infty = \frac{C_\perp V_0}{C(V_\infty)}. \tag{3}$$

Graphically, this means that we have to find the intersection of the curves $C=C(V)$ and $CV=C_\perp V_0=\text{constant}$. The latter equation represents a hyperbola. In the figure, the intersection of these two curves is indicated using another round marker. It is clear that in this example, the voltage drop due to the voltage dependence of the pixel capacitance is enormous.

Another, more practical, case of the voltage drop is illustrated in figure 4. This is a situation that occurs in some implementations of an in-pixel frame buffer. A storage capacitance (20 fF), pre-charged to the desired pixel voltage V_0, is connected and stays connected to the pixel capacitance (see inset). If the pixel capacitance was initially uncharged, the graphical solution is straightforward: again, we construct the intersection between the hyperbola corresponding with the constant initial charge and the (total) $C(V)$ curve. The actual evolution of the voltage takes place in two phases: first, when the switch is closed, there is an immediate charge redistribution between the storage capacitor and the uncharged pixel capacitance, resulting in a jump to the situation (V_1, C_1), with $C_1 = C_{st} + C_\perp$. Next, the pixel starts reacting to the applied voltage V_1 and the total capacitance rises further, while at the same time the voltage drops, until the steady state solution (V_∞, C_∞) is reached. In this case, because of the presence of the storage capacitor, the voltage drop is considerably less, but still by far not negligible. A model for the pixel capacitance must be able to confirm these graphical solutions.

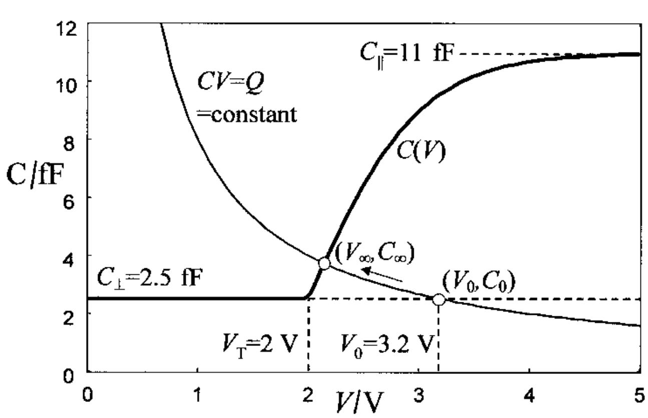

Figure 3. Graphical solution of the voltage drop phenomenon.

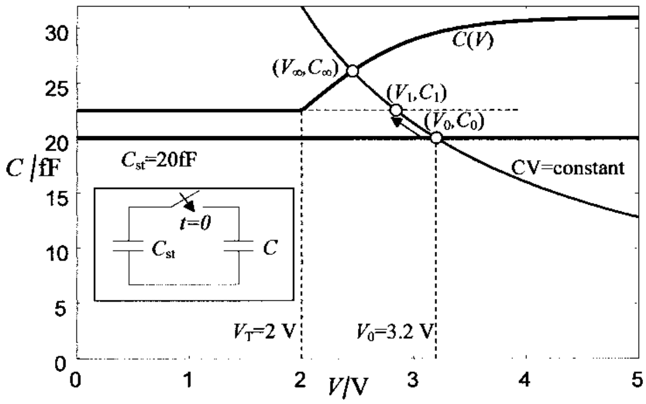

Figure 4. Voltage drop if the pixel is charged from a storage capacitor. At $t=0$, the precharged storage capacitor (3.2 V) is electrically connected to the pixel capacitor.

4. The model

4.1. *Voltage history-dependent capacitance*

Figure 5 shows the basic structure of the proposed pixel model. The voltage between the terminals of the capacitor is squared and fed to a low pass filter formed by R_d and C_d. This models the first-order system with $\mathbf{E}^2$ driving force and $\tau = R_d C_d$ time constant. Then, the square root is taken, resulting again in a voltage, which we will now call the internal voltage V_i. This is in fact the rms (root mean square) voltage over the pixel. Note that V_i^2 plays the role of the one-dimensional variable x that we introduced in §2. Finally, a voltage-driven capacitor is inserted between the two input terminals.

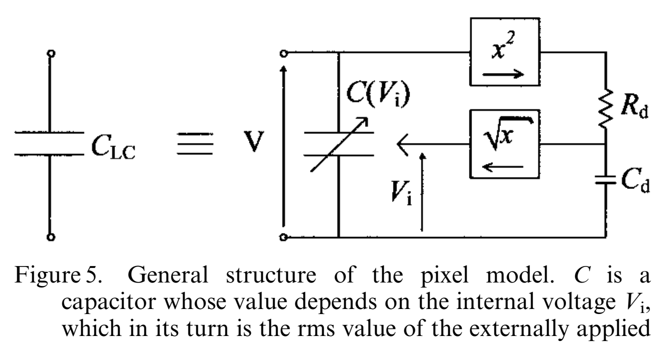

Figure 5. General structure of the pixel model. C is a capacitor whose value depends on the internal voltage V_i, which in its turn is the rms value of the externally applied voltage V.

Its capacitance value is a direct function of the internal voltage V_i.

4.2. *Spice implementation*

In a network simulation program such as PSpice, this voltage-driven capacitance can be implemented as a voltage-dependent current source, with value

$$I = \frac{dQ}{dt} = \frac{d(CV)}{dt} = C(V_i)\frac{dV}{dt} + V\frac{dC(V_i)}{dt}. \quad (4)$$

This Spice implementation is shown in figure 6. It uses a 1-input ABM (analogue behaviour modelling) block with current output.

4.3. *C(V) curve*

Note that the $C(V)$ function in this model must be the same as the quasi-static $C(V)$ curve that can be measured externally, because in a steady state regime, the internal voltage V_i will coincide with the externally applied voltage V. A mathematical function or a reference table can be used to implement $C(V)$ in the Spice model. A simple empirical formula that will suffice for demonstrating the usefulness of our model, is based on the arctan function:

$$C = C_\perp \quad \text{for } V < V_T \quad (5)$$

$$C = C_\perp + \frac{2}{\pi}\left(C_\parallel - C_\perp\right)\arctan\left(\frac{V - V_T}{V_{m,c}}\right) \quad \text{for } V \geq V_T \quad (6)$$

where V_T is the threshold voltage of the pixel and $V_{m,c}$ is the capacitance modulation voltage, which is a measure for the required voltage swing to switch the pixel fully on. Equations (5) and (6) can be written as a single expression, using

$$C = C_\perp + \frac{2}{\pi}\left(C_\parallel - C_\perp\right)\arctan\left(\frac{x + |x|}{2}\right) \quad (7)$$
$$\text{with } x = (V - V_T)/V_{m,c}.$$

The abrupt threshold in this expression can be smoothened by replacing $|x|$ with $\left(x^2 + c^2\right)^{\frac{1}{2}}$, in which c is a dimensionless smoothing constant (technique first used in [6]). Both the unsmoothed and the smoothed (with $c = 0.1$) arctan functions are shown in figure 7. The resulting expression,

$$C = C_\perp + \frac{2}{\pi}\left(C_\parallel - C_\perp\right)\arctan\left[\frac{x + \left(x^2 + c^2\right)^{\frac{1}{2}}}{2}\right] \quad (8)$$
$$\text{with } x = (V - V_T)/V_{m,c}$$

can be fitted very well with $C(V)$ curves generated by simulation software such as DIMOS LCD Workbench from Autronic-Melchers. See, for example, figure 8, in

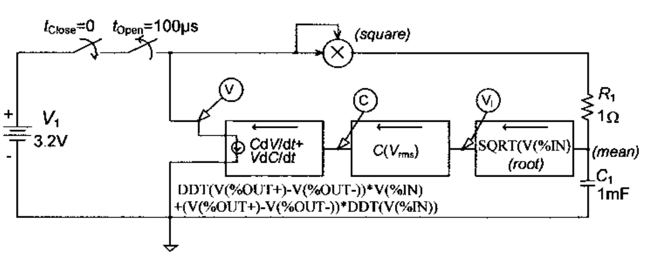

Figure 6. Pspice implementation of the pixel model and the experiment of figures 2 and 3.

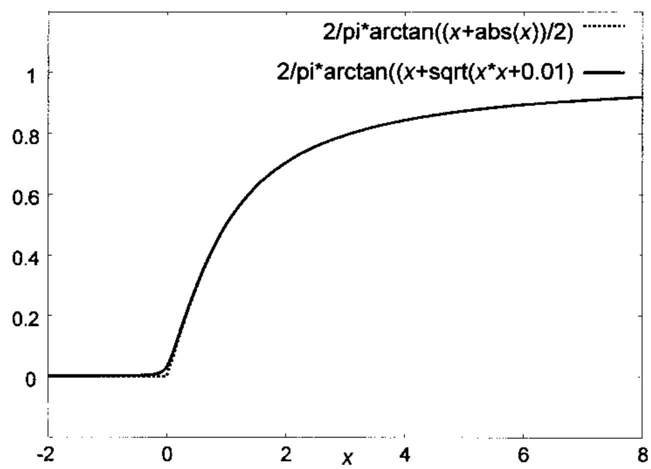

Figure 7. Replacing $x + |x|$ with $x + \left(x^2 + c^2\right)^{\frac{1}{2}}$ smoothens the arctan-based $C(V)$ curve.

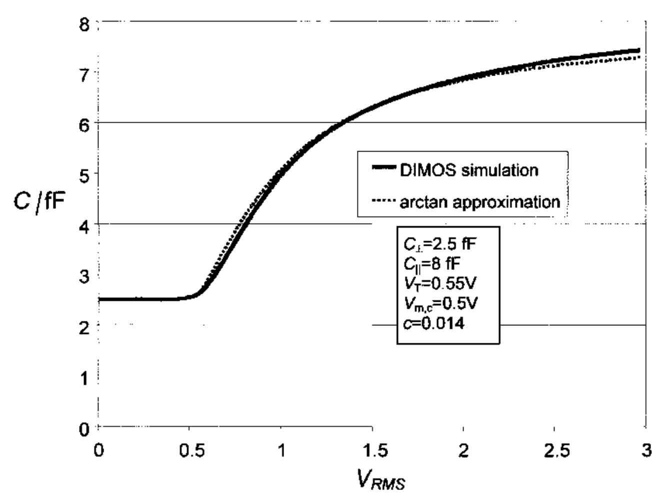

Figure 8. The simple arctan-based $C(V)$ curve can be fitted quite easily to the DIMOS simulation result. DIMOS input data: standard MTC configuration (90°–20°), $n_o = 1.499$, $n_e = 1.6312$, $\varepsilon_o = 4.1$, $\varepsilon_e = 14.1$, $K_{11} = 12.5$ pN, $K_{22} = 7.3$ pN, $K_{33} = 17.9$ pN, $\gamma = 0.1495$ Pa, cell thickness = 1.872 µm.

which the arctan approximation as given by (8) is matched with a DIMOS simulation of a mixed-mode TN (MTN) pixel. We found that equation (8) fits much better to actual $C(V)$ curves than other empirical approximations, such as the $(V/V_T-1)^{\frac{1}{2}}$-based approximation found in [3].

4.4. *Optical response*

Until now, the model has been used to predict the pixel voltage evolution, taking into account the dynamic and voltage history-dependent nature of the pixel capacitance. This is especially useful for predicting the steady state solution of circuits such as those shown in figures 2 and 4. But the model can also be used to predict the dynamic optical response.

It is justified to assume that, as with the pixel capacitance value, the optical response T of the pixel is also a direct function of the internal voltage V_i. Indeed, both are directly related to the LC director distribution inside the cell, which is represented in our model by V_i. Since in the steady state situation, this internal voltage coincides with the externally applied voltage V, the function used for $T(V_i)$ must coincide with the measured quasi-static transmission curve $T(V)$ (or reflection curve in the case of a reflective pixel). A reference table can be used to implement this function in the model, but for ease of calculation, we will use a simple mathematical expression that fits well with experimental data:

$$T = 1-(1-T_{\min})\tanh\left[\frac{y+(y^2+d^2)^{\frac{1}{2}}}{2}\right] \quad \text{with } y=\frac{V_i-V_T}{V_{m,o}}. \tag{9}$$

In this equation d is again a dimensionless smoothing constant, while $V_{m,o}$ is the optical modulation voltage, which is a measure for the voltage swing needed to switch the pixel completely on. Note that due to the different shapes of the $C(V)$ and the $T(V)$ curves, and the different nature of the mathematical approximations used in equations (8) and (9), $V_{m,o}$ is generally different from $V_{m,c}$, and the smoothing constant d is not necessarily the same as c. $T_{\min}$ is the minimum transmission of the pixel and limits the contrast.

Figure 9 illustrates the fit between equation (9) and a $T(V)$ curve measured on a reflective MTN cell. This figure is not a test of our dynamic pixel model, but merely of the mathematical $T(V)$ approximation given by expression (9). For a normally black pixel, expression (9) becomes:

$$T = T_{\min}+(1-T_{\min})\tanh\left[\frac{y+(y^2+d^2)^{\frac{1}{2}}}{2}\right] \quad \text{with } y=\frac{V_i-V_T}{V_{m,o}}. \tag{10}$$

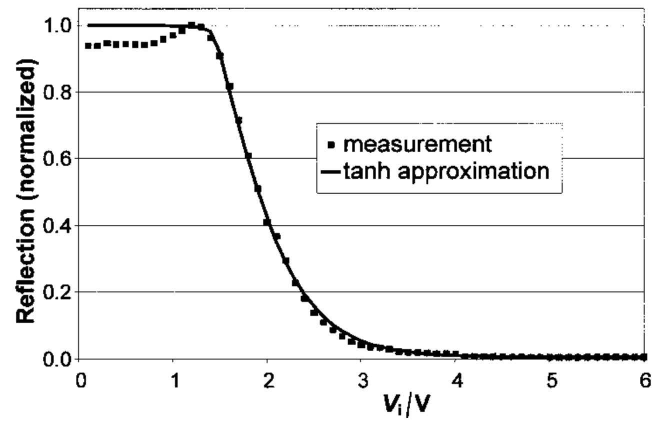

Figure 9. Fitting of the tanh approximation to a measured reflection curve of a mixed-mode TN pixel.

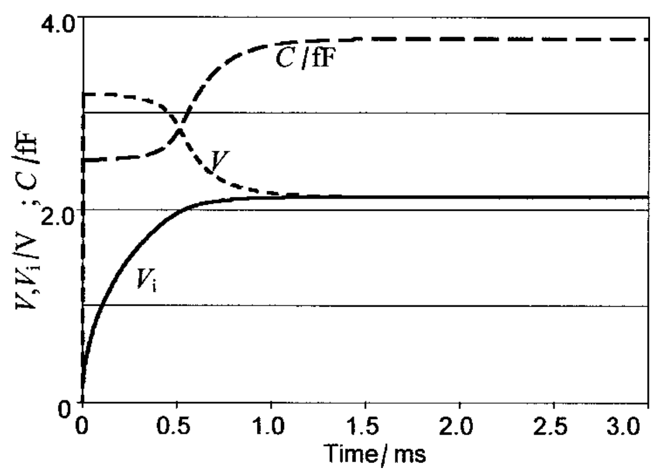

Figure 10. Transient simulation result of the circuit shown in figure 6. The curve C has been scaled so that the vertical axis can be interpreted in fF instead of volts.

5. Results

5.1. *Voltage drop*

We will now try to confirm the graphical solutions found in § 3. Consider again the circuit shown in figure 6. At $t=0$, a voltage of 3.2 V is connected to the pixel capacitance; after 100 μs, the voltage source is disconnected. The transient solution found by PSpice is shown in figure 10; the curves shown correspond with the voltages at markers V_i, C and V in figure 6. The curve C is scaled so that the vertical axis can be interpreted in fF. It is clear that the steady state solution corresponds perfectly with the graphical solution given in figure 3. Note also that in this steady state solution, the external voltage V and the internal voltage V_i are the same, as they should be.

We can also simulate the transient solution of the situation depicted in figure 4. The corresponding Spice circuit is shown in figure 11, and the transient simulation result in figure 12. Also in this case, we obtain a steady state solution that confirms the graphical solution.

5.2. *Dynamic optical behaviour*

The model is suited not only for finding the steady state voltage across the pixel, but also to predict the dynamic behaviour of the pixel. Figure 13 (a) shows the result of such a simulation experiment. A pixel is connected directly to a voltage source switching from 0 to 3, 4 or 5 V and back. In this simulation we used the following device parameters: $R_d C_d = 3$ ms, $C_\perp = 2.5$ fF, $C_\parallel = 8$ fF, $V_T = 2$ V, $V_{m,c} = 0.1$ V, $c^2 = 0.1$, $V_{m,o} = 0.9$ V and $d^2 = 0.1$. The top graph shows the applied voltage

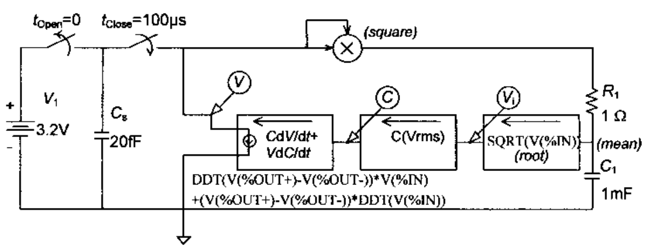

Figure 11. Spice circuit corresponding with the voltage drop experiment of figure 4.

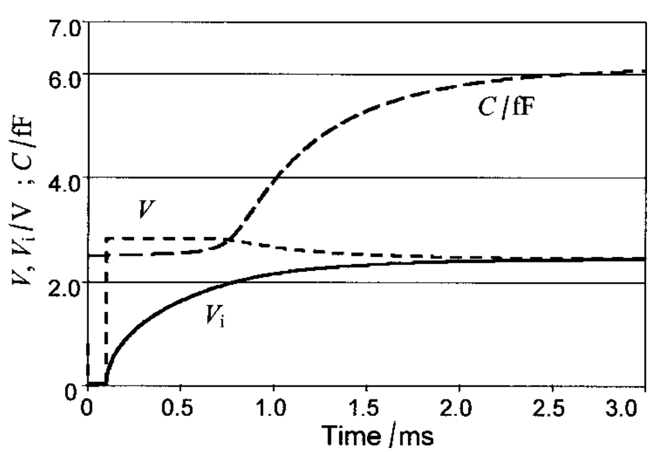

Figure 12. Transient simulation result of the circuit shown in figure 11. Curve C has again been scaled so that the vertical axis reads in fF instead of volts.

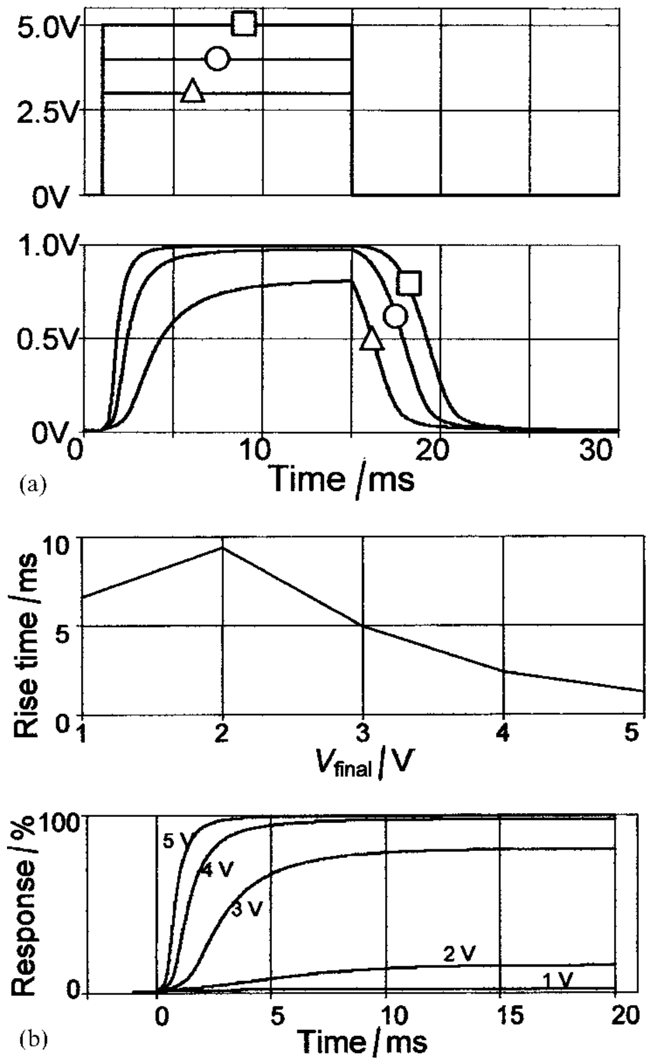

Figure 13. Transient simulation result of a pixel connected to a voltage source. (a) The top graph shows the applied voltages. The bottom graph shows the resulting optical responses. (b) The top graph shows the rise time (10–90%) of the optical response plotted as a function of final voltage: the initial voltage is always 0V. The bottom graph shows the actual transient simulation results from which rise times are determined.

waveforms, and in the bottom graph the corresponding optical responses for a normally black LC mode are plotted. From this experiment, it is clear that the model is capable of predicting the voltage-dependent rise times and almost voltage-independent fall times that are typical for most nematic liquid crystal pixels.

Elaborating this experiment further, figure 13(b) shows the calculated rise time of the optical response (10–90%, normally black mode) as a function of the final voltage, the initial voltage being kept at zero. Clearly, the rise time decreases with increasing voltage

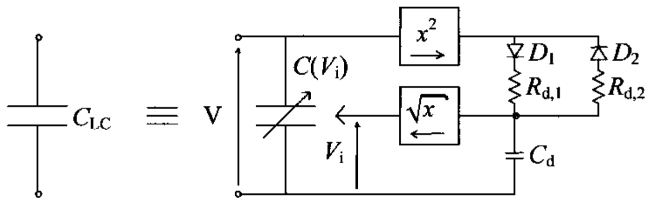

Figure 14. The addition of two diodes and a second resistor to the RC filter gives individual control over the pixel rise and fall times.

stimulus. This behaviour is consistent with the experimental data presented in, for example, [7].

6. Discussion and possible enhancements

The validity of the model we have presented depends, of course, on the validity of the assumptions made while constructing it. Perhaps the most important assumption is that, at least for the sake of electrical simulation, the status of a liquid crystal pixel can be represented by an internal status variable (x or V_i) that depends solely on the voltage history. A further crucial assumption is that the simple first-order mechanical model of figure 1 is applicable. This model implies that the internal variable has a root-mean-square response to the applied pixel voltage. Fortunately, this is consistent with existing LC models, at least for simple nematic modes. For those cases where the mechanical model of figure 1 is invalid, generalizations of the model are possible. We can, for example, replace the first-order low pass filter formed by R_d and C_d by a higher-order filter. Also, it is possible to add a second resistor and two diodes to this RC filter, as shown in figure 14. This gives individual control over the pixel rise and fall behaviour.

7. Conclusions

Starting from a simple first-order mechanical model for the average director orientation inside a liquid crystal pixel, it was possible to generate a Spice macro model that describes its voltage history-dependent behaviour. The model comprises an internal node with a voltage V_i that we call the internal voltage of the pixel. V_i depends on the voltage history in a very simple way: it is nothing more than the root-mean-square value of the pixel voltage, calculated with an integration time $\tau = \gamma / K = R_d C_d$ that depends on the viscosity γ and elastic anchoring constant K of the liquid crystal molecules. From the internal voltage, not only the momentary capacitance value, but also the optical response can be calculated, using the quasi-static $C(V)$ and $T(V)$ curves. The structure of the model is flexible and allows for modifications and generalizations such as higher-order input voltage filtering or asymmetric integration time constants.

The model accurately predicts the voltage drop that occurs in an isolated pixel due to internal realignment of the liquid crystal. Furthermore, dynamic simulation results of the optical response are in agreement with experimental data. Therefore, the model can be used in detailed simulations of the pixel behaviour in active matrix architectures without, or with only small, storage capacitors, such as the emerging smart pixel architectures.

This work was carried out with support from the European Commission under the Framework 5 program, project LCOS4LCOS, N° IST-2001-34591.

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